Calculus

Course booklet

Mathematics for university students. Contains polynomials, trigonometric functions, sequences and series, differentiation and more.

SOWISO

ABOUT & PRICING

About SOWISO

SOWISO offers:

- a homework, practice and **learning environment**;
- personalised **feedback** on all answer attempts;
- different **testing and assessment** tools;
- customisable **mathematics courses** with explanations, examples, and endless **randomised practice exercises**;
- an authoring tool to **create original material**;
- **learning analytics** giving detailed insight into student performance;
- **integration** with your LMS/VLE.

Our learning environment guides students along as they solve problems. When doing exercises, students can enter open answer calculations or mathematical formulas. The software will analyse their answer and provide targeted feedback and hints helping the student understand the next step in the solution process, and/or highlight any mistakes they made.

SOWISO increases student engagement and saves teachers time checking and grading!

Pricing

SOWISO partners with higher education institutions on a SAAS licensing basis.

The cost for the platform starts at € 5.50 per student per year, with an additional per student per year fee of € 7.50 per course.

A second licensing model is one in which students pay for their own license in our webshop.

Our digital courses are a fully interactive alternative for paper books and offer a personalised and adaptive learning experience that fits today's generation of students.

How are courses structured?

The courses are structured in chapters and subchapters consisting of units. The unit subjects are listed in more detail on the following pages.

Each unit consists of (at least) one theory page and one package of exercises.

Theory pages contain explanations, (randomised) examples and visualisations and (interactive) graphs.

The packages of **exercises** contain on average around 10 exercises. Each of these exercises are randomised, allowing for endless practicing, and include targeted hints and personalised feedback for the students while solving the exercises.

COURSE CONTENT

Chapter 1: Functions (14 topics)

1. *Sets (3 topics)*
    a. The notion of sets
    b. Operations for sets
    c. Intervals
2. *Functions (2 topics)*
    a. The notion of function
    b. Operations for functions
3. *Range (4 topics)*
    a. The range of a function
    b. Functions and graphs
    c. Transformations of the axes
    d. Symmetry of functions
4. *Injectivity (4 topics)*
    a. Injective functions
    b. The inverse of a function
    c. Power functions
    d. Equations and functions
5. *Applications (1 topic)*
    a. Applications of functions

Chapter 2: Polynomials & rational functions (18 topics)

6. *Polynomials (3 topics)*
    a. The notion of polynomial
    b. Calculating with polynomials
    c. Division with remainder for polynomials
7. *Linear polynomials (1 topic)*
    a. Linear functions

8. *Quadratic polynomials (3 topics)*
    a. Quadratic functions
    b. Quadratic equations
    c. Quadratic inequalities

9. *Factorization of polynomials (7 topics)*
    a. The notions gcd and lcm for polynomials
    b. Rules of calculation for gcd and lcm of polynomials
    c. The Euclidean algorithm for polynomials
    d. Factorization of polynomials
    e. The fundameltal theorem of algebra
    f. Polynomial interpolation
    g. The extended Euclidean algorithm for polynomials

10. *Rational functions (3 topics)*
    a. The notion of rational function
    b. Normal form for rational functions
    c. Partial fraction decomposition for rational functions

11. *Applications (1 topic)*
    a. Applications of polynomials and rational functions

Chapter 3: Trigonometric functions (9 topics)

12. *Basics (3 topics)*
    a. Definitions of sin and cos
    b. Right triangles and trigonometric functions
    c. Periodicity of trigonometric functions

13. *Calculation (3 topics)*
    a. Special values of trigonometric functions
    b. Addition formulas for trigonometric functions
    c. Triangles and trigonometric functions

14. *More trigonometric functions (2 topics)*
    a. Tangent and cotangent
    b. Inverse trigonometric functions

15. *Applications (1 topic)*
    a. Applications of trigonometric functions

Chapter 4: Exponential & logarithmic functions (8 topics)

16. *Definition exp (3 topics)*
    a. The notion of exponential function
    b. Rules of calculation for exponential functions
    c. Equations with exponential functions

17. *Definition log (3 topics)*
    a. The notion of logarithm
    b. Rules of calculation for logarithms
    c. Equations with logarithms

18. *Growth (1 topic)*
    a. Exponential growth

19. *Applications (1 topic)*
    a. Applications of exponential and logarithmic functions

Chapter 5: Limits (11 topics)

20. *Definition (4 topics)*
    a. The notion of limit
    b. The notion of limit and infinity
    c. Limits of rational functions
    d. Vertical asymptotes

21. *Rules for calculating limits (4 topics)*
    a. Rules for limits
    b. Horizontal asymptotes
    c. Oblique asymptotes
    d. Squeeze theorem for limits

22. *Exp and gonio (2 topics)*
    a. Limits of exponential functions
    b. Trigonometric limits

EXERCISE & THEORY EXAMPLE

Feedback

Calculate the derivative $h'(x)$ of $h(x) = 7 \cdot \sin(8 \cdot x)$.

$h'(x) = 7 \cdot \cos(8 \cdot x)$ — No, you may have forgotten to multiply by $g'(x)$, in which $g(x) = 8 \cdot x$.

$h'(x) = 7 \cdot \cos(56 \cdot x)$ — The chain rule indicates that the derivative contains $f'(g(x))$, in which $f(x) = 7 \cdot \sin(x)$ and $g(x) = 8 \cdot x$. Thus, the the argument of the cosine must be equal to $g(x)$. This is not the case in your answer.

$h'(x) = 56 \cdot \cos(8 \cdot x)$ — Great job

Practise example

1 2 3 4 5 6 7

Multivariate Functions: Stationary points

Minimum, maximum, and saddle

The function

$$f(x,y) = -3$$

has a local maximum. Which point is it?

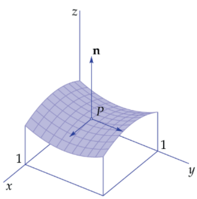
[,]

Check

Example

Below is the graph of the function

$$f(x,y) = \frac{1}{2}\left((1-(x-\tfrac{1}{2})^2 + (y-\tfrac{1}{2})^2\right)$$



This function has the following partial derivatives:

$$f_x(x,y) = \frac{1}{2} - x \qquad \text{and} \qquad f_y(x,y) = y - \frac{1}{2}$$

At the point $\frac{1}{2}, \frac{1}{2}$ both derivatives are zero again. But the coordinate line in the direction of the x-axis is a downward parabola, and the coordinate line in the direction of the y-axis is an upward parabola. Therefore, the point $\left[\frac{1}{2}, \frac{1}{2}\right]$ must be a saddle point.

Example

The function

$$f(x,y) = -3 \cdot x^2 + 2 \cdot y \cdot x - 9 \cdot y^2 + 14$$

has a local maximum. Which point is it?

Solution

$[0,0]$

Since a local maximum of a multivariate differentiable function is

Theory example

23. *Applications (1 topic)*
a. Applications of limits

Chapter 6: Sequences & series (11 topics)

24. *Definition (3 topics)*
a. The notions of sequence and series
b. Arithmetic series
c. Geometric series

25. *Convergence (3 topics)*
a. Convergence
b. Monotonic sequences
c. Divergence

26. *Rules (1 topic)*
a. Rules for limits of sequences

27. *Power series (2 topics)*
a. Power series
b. Convergence criteria

28. *Length (1 topic)*
a. Length

29. *Applications (1 topic)*
a. Applications of sequences and series

Chapter 7: Continuity (8 topics)

30. *Definition of continuity (3 topics)*
a. The notion of continuity
b. Global minimum and maximum
c. Continuous extension

31. *Min-max and Intermediate Value Theorem (2 topics)*
a. The Min-Max Theorem
b. Intermediate Value Theorem

32. *Limits (2 topics)*
   a. Limits of continuous functions
   b. Rules for continuity

33. *Applications (1 topic)*
   a. Applications of continuity

Chapter 8: Differentiation (14 topics)

34. *Definition (3 topics)*
   a. The notion of difference quotient
   b. The notion of differentiation
   c. A simple derivative

35. *Simple rules (4 topics)*
   a. The derivative of a sum function
   b. The derivative of a polynomial
   c. The product rule for differentiation
   d. Tangent lines

36. *More rules (4 topics)*
   a. The chain rule for differentiation
   b. Derivatives of trigonometric functions
   c. The quotient rule for differentiation
   d. Derivatives of inverse functions

37. *Exp and log (2 topics)*
   a. The natural logarithm
   b. Derivatives of exponential and logarithmic functions

38. *Applications (1 topic)*
   a. Applications of differentiation

Chapter 9: Analysis of functions (10 topics)

39. *Minima and maxima (3 topics)*
   a. Local minima and maxima
   b. The Mean Value Theorem

c. Monotonocity

40. *Higher derivatives (1 topic)*
    a. Higher derivatives

41. *Implicit derivatives (1 topic)*
    a. Implicit derivatives

42. *Approximation with polynomials (3 topics)*
    a. Linear approximation
    b. Taylor series
    c. Taylor series of some known functions

43. *De L'Hôpital (1 topic)*
    a. The De L'Hôpital rule

44. *Applications (1 topic)*
    a. Applications of analysis of functions

Chapter 10: intergration (11 topics)

45. *Antiderivation (3 topics)*
    a. The notion of an antiderivative
    b. Antiderivatives of some known functions
    c. Integration by parts

46. *Area (1 topic)*
    a. Area

47. *Integral (3 topics)*
    a. Riemann sums
    b. The integral of a function
    c. Rules of calculation for integrals

48. *Estimates (2 topic)*
    a. Estimates of integrals
    b. Mean Value Theorem for Integrals

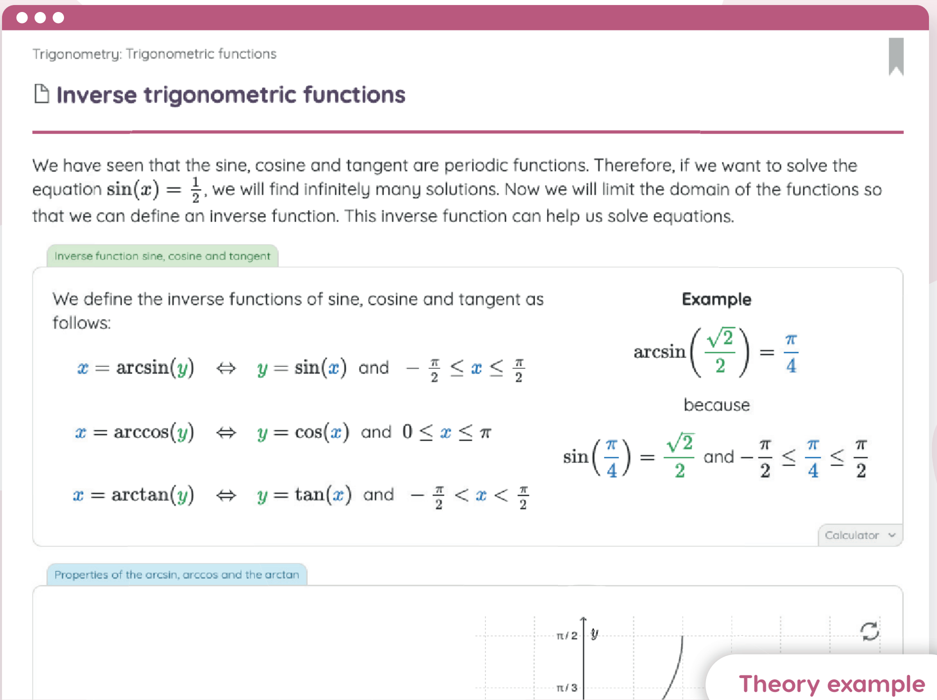
49. *The Fundamental Theorem of Calculus (1 topic)*
   a. The fundamental theorem of calculus

50. *Applications (1 topic)*
   a. Applications of integration

We are currently expending our Calculus course with two new chapters: Logic and Set Theory

Missing something? SOWISO allows teachers to create their own content in our authoring environment.

THEORY EXAMPLE



Theory example

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